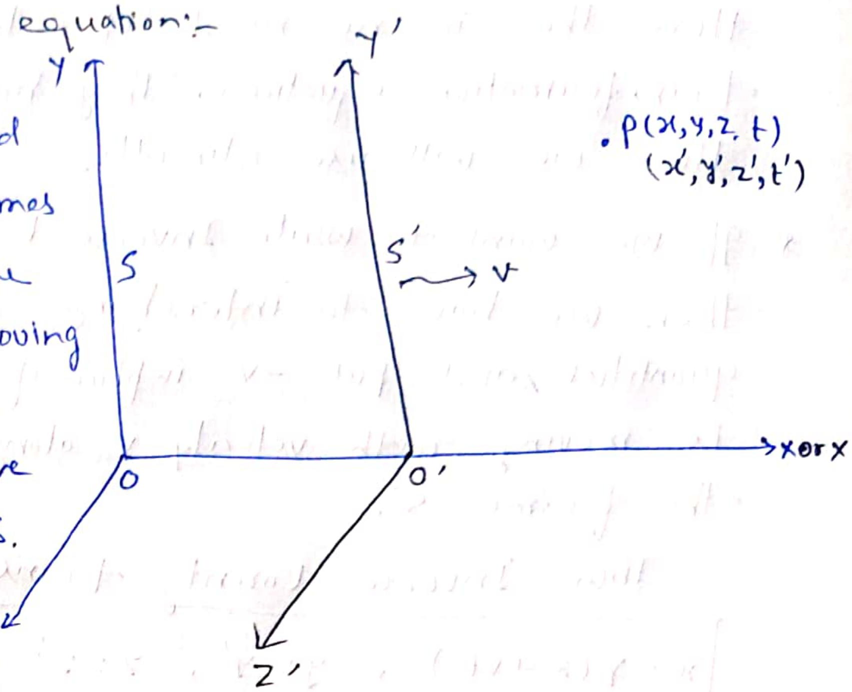
For under-standing Relativistic Electrodynamics, some basic knowledge of special theory of relativity should be in memory.

* Velocity of light in all inertial frame will be same.

* Lorentz transformation equation:-



In shown figure, S and S' are two inertial frames of reference in which the inertial frame S' is moving with constant velocity v along +ve x axis relative to the inertial frame S.

Suppose an event occurs at point P whose coordinates observed from S and S' are (x, y, z, t) and (x', y', z', t') respectively.

Here $t = t' = 0$ when their origins coincide.

If the coordinate (x, y, z, t) of the point P as seen from the inertial frame S be known then coordinates (x', y', z', t') of the point P as seen from the frame S' can be obtained by using Lorentz transformation equations as follows.

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x - vt}{\sqrt{1 - \beta^2}} = \gamma(x - vt)$$

$$y' = y, \quad z' = z$$

and $$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \beta^2}} = \gamma\left(t - \frac{v}{c^2}x\right)$$

when $\beta = \frac{v}{c}$

$\gamma = \sqrt{1 - \beta^2} = \sqrt{1 - \frac{v^2}{c^2}}$.

Thus Lorentz transformation equations are

$$\boxed{x' = \gamma(x - vt), \quad y' = y, \quad z' = z \quad \text{and} \quad t' = \gamma\left(t - \frac{v}{c^2}x\right)} \quad \text{(A)}$$

Here relative motion between the two frames S and S′ is along x axis so all distances perpendicular to x axis remains unchanged. This is why $y' = y$ and $z' = z$.

Here there is no need of derivation of Lorentz transformation equations. They have been derived in TDC part 1. Here we will use directly.

* If we want to write Inverse Lorentz transformation equation then we have to interchange primed and unprimed quantities and put $-v$ in place of v because the frame S is moving with velocity v along $-$ve x axis relative to the frame S′.

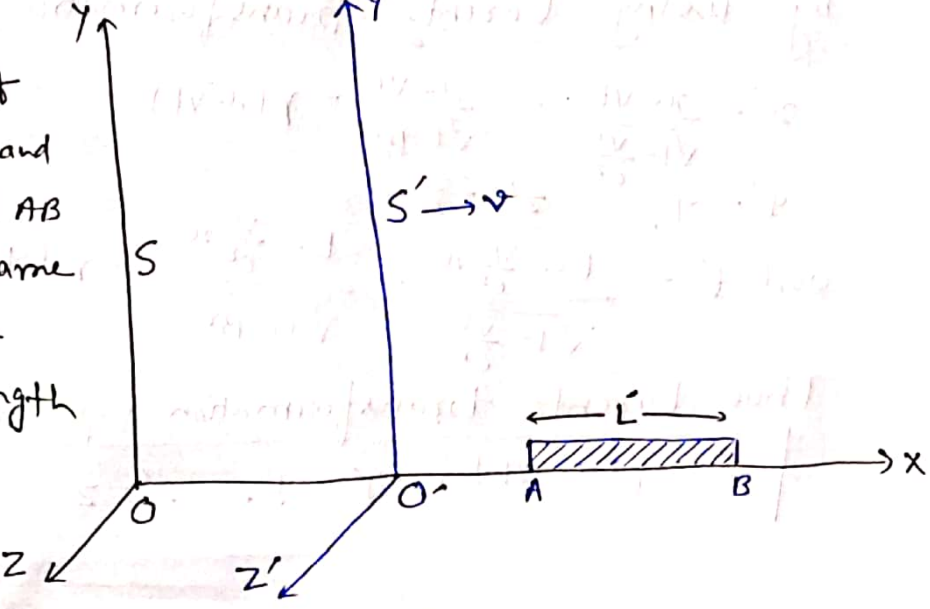
Thus Inverse Lorentz transformation equation will be

$$\boxed{x = \gamma(x' + vt'), \quad y = y', \quad z = z' \quad \text{and} \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right)} \qquad (B)$$

Therefore, if coordinate (x', y', z', t') of the point P as seen from the frame S′ be known then the coordinate (x, y, z, t) of the point P as seen from the frame S can be obtained by using Inverse Lorentz transformation equation.

* Let us see some important results (consequences) of these equations.

** Length contraction :- If the x-coordinates of A and B two ends of the rod AB observed from the frame S at the same t be x_1 and x_2 then length of the rod as seen



from the frame S will be

$$L = x_2 - x_1.$$

Here the rod AB is placed in the frame S′.

From Lorentz transformation equation

$$x_1' = \gamma(x_1 - vt) \quad \text{and} \quad x_2' = \gamma(x_2 - vt)$$

Here t_1 and t_2 are same as t but $x_1 \neq x_2$ in frame S

so $t_1' \neq t_2'$ in the frame S′.

Now length of the rod AB as seen from the frame S′ will be

$$L' = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt)$$

$$= \gamma(x_2 - x_1)$$

$$L' = \gamma . L \Rightarrow L' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} L \qquad \because \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{L = \sqrt{1 - \frac{v^2}{c^2}} . L'} \quad \text{or} \quad L < L'.$$

Since the rod AB is placed in the frame S′, so its length in the frame S′ will be same L' but its length will appear $L = \sqrt{1 - \frac{v^2}{c^2}} . L'$ as seen from the frame S.

Thus length of the rod will be contracted by a factor of $\sqrt{1 - \frac{v^2}{c^2}}$ as seen from S.

If $v = c$ then $L = \sqrt{1 - \frac{c^2}{c^2}} . L' = 0$

Thus if the frame S′ is moving with velocity c equal to velocity of light relative to S and the rod AB is placed in S′ then the rod AB will appear as a point as seen from S because length becomes zero.

** Time dialation :- If two events occur at same position x' in the frame S′ at times t_1' and t_2' then the time interval between two events occuring at position x' in frame S′ will be

$$\Delta t' = t_2' - t_1'$$

If t_1 and t_2 be the times measured for the two events in the frame S then time interval between the two events in the frame S will be

$$\Delta t = t_2 - t_1$$

From inverse Lorentz transformation equation

$$t_1 = \gamma\left(t_1' - \frac{v}{c^2}x'\right) \quad \text{and} \quad t_2 = \gamma\left(t_2' - \frac{v}{c^2}x'\right)$$

$\because x_1' = x_2' = x'$ same position in S'.

Now
$$t_2 - t_1 = \gamma\left(t_2' - \frac{v}{c^2}x'\right) - \gamma\left(t_1' - \frac{v}{c^2}x'\right)$$

$$= \gamma(t_2' - t_1')$$

$$\Delta t = \gamma \cdot \Delta t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \Delta t' \qquad \because \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Therefore the time interval Δt is dialated by a factor of $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ in comparision with proper time interval $\Delta t'$, ie, a stationary observer in the frame S measures longer time interval between the two events occuring in the frame S'. In other words, a moving clock with frame S' appears to be slowed down to a stationary observer in the frame S. This phenomena is known as time dialation.

** Relativistic mass variation :- If a body of mass m_0 is placed in frame S' moving with velocity v relative to the frame S, then mass of the body measured from a stationary observer in the frame S will be

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

m_0 = rest mass of the body
m = kinetic mass of the body.